



The Federal University of Technology, Akure
School of Engineering and Engineering Technology
Department of Electrical and Electronic Engineering
First Semester Examinations 2013/2014 Session

EEE 405 Communications Principle .

Date: June 2014.

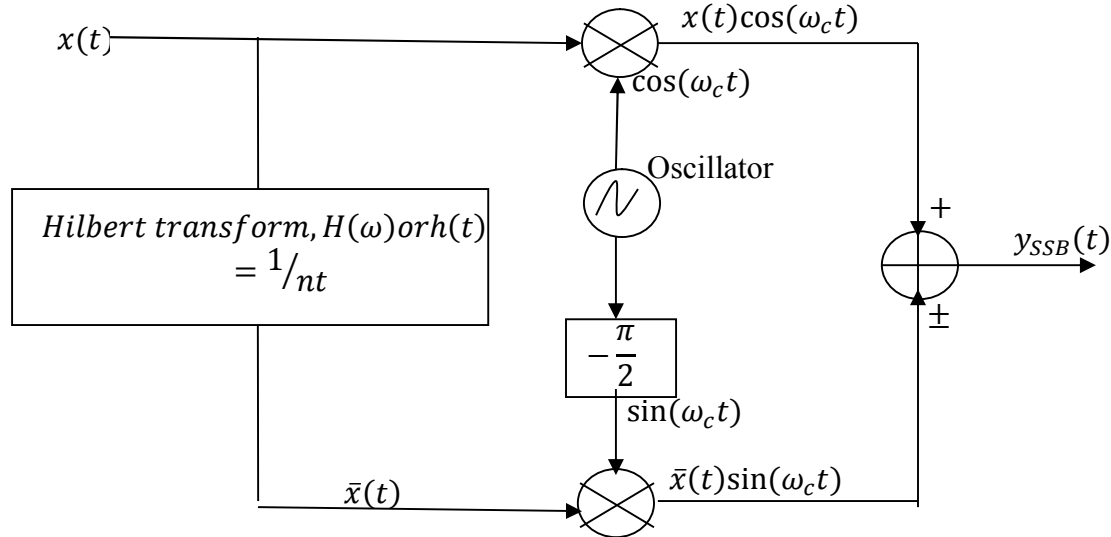
Instructions:

Attempt Any Four Questions .

Time Allowed:

2 ½ HOURS.

- 1a As a Communication Consultant, a client came for advice on the approach to transmit with minimal energy wastage and narrower frequency band; using the phase shift method, explain the generation of the SSB modulated signal



Generation of Single sideband by phase-shifting process

Phase-shift method of single-sideband generation is direct implementation of

$y_{SSB}(t) = \frac{A_c}{2} [x(t)\cos(\omega_c t) \pm \bar{x}(t)\sin(\omega_c t)]$ which is represented as shown above.

In the figure:

- Top multiplier yields the first product of the DSB term (assuming $A_c = 1$), i.e. $x(t)\cos(\omega_c t)$
- Whilst the bottom multiplier yields the second product term after it has undergone phase shift and the input signal via Hilbert transformer-the 90° ($\pi/2$) phase shifter

Both the top and bottom terms are passed through an output adder yielding the single sideband output, i.e. $y_{SSB}(t)$

Unlike the filtering method, the phase-shift method does not require band pass filters. In practice, it is difficult to build a wideband phase shifting Hilbert transform network that offers a uniform phase shift of -90° over the entire message bandwidth [Thomas and Sekhar, 2005].

However, it is possible to design a practical system that accepts message signal $x(t)$ as input and yields two signals $x_1(t)$ and $x_2(t)$ at its output, which are equal in magnitude and are *orthogonal* to each other, which could then be fed to the multipliers. The SSB is bandwidth efficient compared to the conventional AM, and could be used for several purposes.

b) Show that the bandwidth at resonant frequency

$$\omega_2 - \omega_1 = \frac{\omega_o}{Q}$$

Sketch the amplitude characteristic showing the 3dB point.

Normalized Response

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right) \quad (1)$$

At resonance

$$Z_o = R = \frac{1}{Y_o} \quad (14)$$

The dimensionless ratio of the admittance at any frequency ω to that at the resonant frequency ω_o is

$$\frac{Y}{Y_o} = \frac{Z_o}{Z} = \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \frac{1}{1 + j\left(\frac{\omega L}{R} - \frac{1}{\omega CR}\right)} \quad (2)$$

i.e

$$\begin{aligned} \frac{Y}{Y_o} &= \frac{R}{R + j\left(\omega L - \frac{1}{\omega C}\right)} = \\ &= \frac{1}{1 + j\left(\frac{\omega L \omega_o}{R \omega_o} - \frac{1}{\omega CR \omega_o}\right)} \quad (3) \\ &= \frac{1}{1 + jQ\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right)} \quad (4) \end{aligned}$$

Bandwidth

$$\left(\frac{\omega}{\omega_o} - \frac{\omega_o}{\omega}\right) = \pm \frac{1}{Q} \quad (5)$$

$$\frac{\omega_1}{\omega_o} - \frac{\omega_o}{\omega_1} = -\frac{1}{Q} \quad (6a)$$

$$\frac{\omega_2}{\omega_o} - \frac{\omega_o}{\omega_2} = \frac{1}{Q} \quad (6b)$$

$$\omega_1 = \omega_o \sqrt{1 + \left(\frac{1}{2Q}\right)^2} - \frac{\omega_o}{2Q} \quad (7a)$$

$$\text{and } \omega_2 = \omega_o \sqrt{1 + \left(\frac{1}{2Q}\right)^2} + \frac{\omega_o}{2Q} \quad (7b)$$

$$\omega_2 - \omega_1 = \frac{\omega_o}{Q} \text{ or } f_2 - f_1 = \frac{\omega_o}{Q} \quad (8)$$

Then the 3dB diagram

2a) Explain briefly the processes listed below:

- i) Quantization
- ii) Pulse amplitude modulation

PAM is a special form of analogue pulse modulation. In analogue system, the signal is smoothly varying (sinusoid). A digital system, on the other hand, quantizes or breaks analogue signal into prescribed number of discrete amplitude levels—like an ascending and descending staircase, as demonstrated in Fig 8. If the quantized signals are transmitted as pulses of varying heights, the resultant system could be a modified form of PAM.

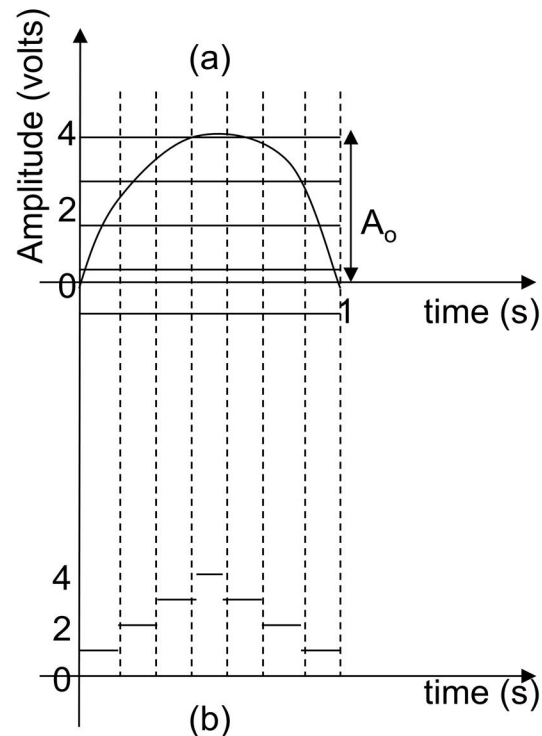


Figure 8. Quantization process: (a) original analogue signal of amplitude A_o ; and (b) quantized version of the signal.

- iii) Pulse code modulation

The quantized signal can be coded in binary coded decimal (BCD) representation where each decimal digit is converted to its 4-bit pure binary equivalent, as in Fig 9. Binary means "two," or "base two." The binary system is a way of counting using just the two numbers: 0 and 1. A system in which the standard values of a quantised wave are indicated by '1s' and '0s' coded

signals is called *pulse-code modulation* (PCM) system. There are two standards of PCM systems, namely: the North American 24-channel system (T1 carrier standard), or the European (E1 carrier standard) or ITU equivalent 30-channel system. Although the two systems employ basically the same principles, they use different philosophies as far as signalling is concerned. ITU (International Telecommunications Union) is an organization whose purpose is to promote international cooperation in the use and improvement of telecommunications of all kinds. Founded in Paris in 1865 as the International Telegraph Union, it became an agency of the United Nations in 1947. You can check ITU's website (www.itu.int) for a variety of information on communications issues.

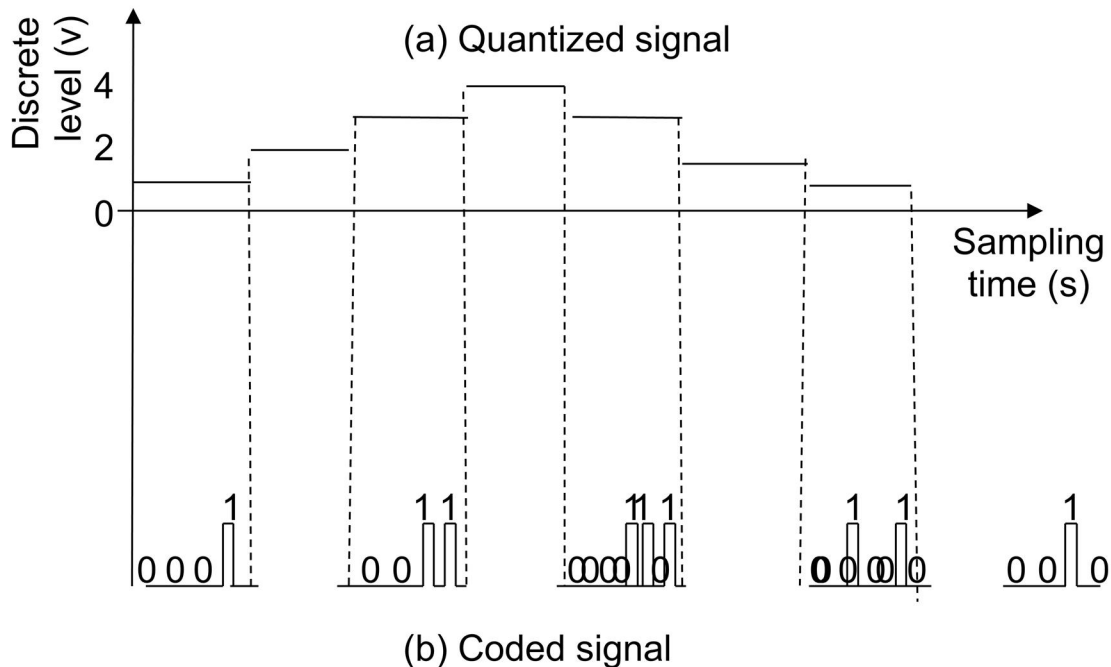


Figure 9. Quantization and Coding process.

Pulses representing the sample values of a PCM waveform can be transmitted on an RF (radio frequency) carrier by the use of amplitude, phase, or frequency. For instance,

1. Amplitude: amplitude shift keying (ASK), where the carrier is determined by the data bit for the interval.
2. Phase: phase shift keying (PSK), where the carrier phase is established by the data bit.
3. Frequency: frequency shift keying (FSK), where the carrier frequency is established by the data bit.

These techniques are digital modulation schemes.

iv) Nyquist rate

Technically, in PAM, the amplitude of the carrier consisting of a periodic train of rectangular pulses is varied in proportion to sample values of a range signal. In this type, pulse duration is held constant. To sample the original signal, and if we are to reduce quantisation error, we must sample twice the rate of the bandwidth, i.e., $2f_m$. This sampling rate is called *Nyquist rate*.

If each quantized signals carrier $\log_2 S$ bits of information, the system capacity, C , or rate of information transmitted must be

$$C = 2f_m \log_2 S \quad \text{bits per second (b/s)} \quad (1)$$

The effect of noise is not factored into this expression. Shannon (1949) showed that the maximum possible rate of transmission of binary digits with bandwidth $B (=2f_m)$ and considering the effect of noise, N , in the transmission channel could be expressed as

$$C = B \log_2 \left(1 + \frac{S}{N} \right) \quad \text{b/s} \quad (2)$$

v) Sampling interval

3a) A carrier wave represented by $1.25\cos(2\pi f_c t)$ (volts) is amplitude modulated by a second wave represented by $0.5\cos(2\pi f_m t)$ (volts). If $f_c = 1\text{MHz}$ and $f_m = 1\text{kHz}$, Calculate the:

- i) Lower and upper side frequencies
- ii) Amplitude of the side frequencies
- iii) Fraction of the power transmitted in the sidebands

Solution;

Carrier wave = $1.25\cos(2\pi f_c t)$ (volts)

Amplitude modulated = $0.5\cos(2\pi f_m t)$ (volts).

$f_c = 1\text{MHz}$ and

$f_m = 1\text{kHz}$,

- i) Lower side frequencies (LSB) = $f_c - f_m$
 $= 1000000 - 1000$
 $= 999000 = 999\text{kHz}$

$$\begin{aligned} \text{Upper side frequencies (USB)} &= f_c + f_m \\ &= 1000000 + 1000 \\ &= 1001000 = 1001\text{kHz} \end{aligned}$$

- ii) Amplitude of the side frequencies

$m = \text{modulating index}$

$E_c = \text{Carrier wave}$

$$A = \frac{mE_c}{2} \quad (1)$$

$$\text{Meanwhile } m = \frac{B}{A}$$

$$= \frac{0.5}{1.25}$$

$$= 0.4$$

Substituting values into eq. (1)

$$A = \frac{mE_c}{2} = \frac{0.4 * 1.25}{2}$$

$$= 0.25 V_c$$

$$\text{iii) } P_t = P_c + P_{LSB} + P_{USB}$$

The instantaneous voltage waveform is:

$$V = V_c \sin \omega_c t + \frac{mV_c}{2} (\cos(V_{oc} - V_{om}) + \frac{mV_c}{2} (\cos(V_{oc} + V_{om}))$$

$$P_c = \frac{\left(\frac{V_c}{\sqrt{2}}\right)^2}{R}$$

$$= \frac{\left(\frac{V_c}{\sqrt{2}}\right)^2}{R} = \frac{V_c^2}{2R} \text{ Power in the Carrier}$$

$$P_{LSB} = P_{USB}$$

$$P_{LSB} = P_{USB} = \frac{\left(\frac{mV_c/2}{\sqrt{2}}\right)^2}{R} P$$

$$\frac{m^2 V_c^2 / 2^2}{(\sqrt{2})^2}$$

$$= \frac{R}{m^2 V_c^2 / 4 * \frac{1}{2}}$$

$$= m^2 V_c^2 / 4 * \frac{1}{2R}$$

$$= \frac{m^2}{4} * \frac{V_c^2}{2R}$$

$$= \frac{m^2}{4} * P_c$$

$$P_t = P_c + 2P_{LSB}$$

$$P_t = P_c + 2 \frac{m^2}{4} * P_c$$

$$\begin{aligned}
P_t &= P_c + \frac{m^2}{2} * P_c \\
&= \left(1 + \frac{m^2}{2}\right) * P_c \\
\frac{P_t}{P_c} &= 1 + \frac{m^2}{2} \\
&= 1 + \frac{(0.4)^2}{2} \\
&= 1 + \frac{0.16}{2} \\
&= 1 + 0.08 = 1.08W
\end{aligned}$$

4a) The figure Q4 below is a time-variant signal $f(t)$ with the fundamental period and amplitude as stated in the diagram,

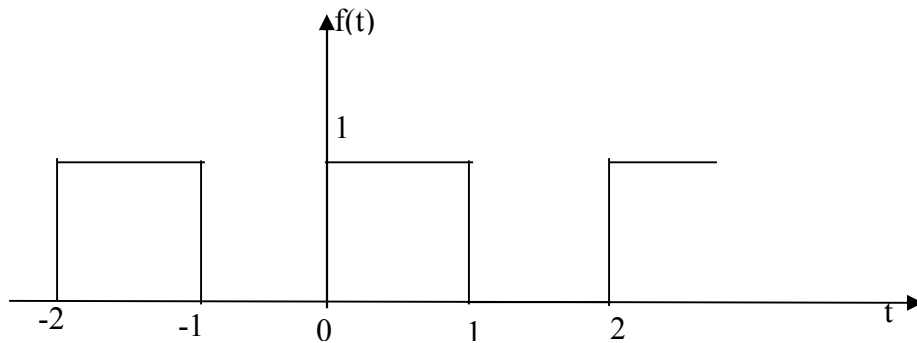


Fig. Q4;

- i) Obtain the signal's Fourier transform
- ii) phase spectra of the signal.

Solution:

$$F(t) = \begin{cases} 1 & 0 < t < 1 \\ 0 & 1 < t < 2 \end{cases}$$

$$F(t) = a_o + \sum_{n=1}^{\infty} a_n \cos n\omega_o t + b_n \sin n\omega_o t$$

Since

$$T = 2, \quad \omega_o = 2\pi f$$

Where

$$f = \frac{1}{T}, \quad \omega = \frac{2\pi}{T}, \quad \omega = \frac{2\pi}{2} = \pi$$

$$\begin{aligned}
a_o &= \frac{1}{T} \int_0^1 f(t) dt \\
a_o &= \frac{1}{2} \int_0^1 dt + \frac{1}{2} \int_1^2 0 dt \\
&7
\end{aligned}$$

$$a_o = \frac{1}{2} \int_0^1 dt$$

$$a_o = \frac{1}{2} [t]_0^1$$

$$a_o = \frac{1}{2}$$

$$a_n = \frac{2}{T} \int_0^2 f(t) \cos n\pi_t dt$$

$$a_n = \frac{2}{2} \left(\int_0^1 1 \cos n\pi_t dt + \int_1^2 0 \cos n\pi_t dt \right)$$

$$a_n = \int_0^1 1 \cos n\pi_t dt + \int_1^2 0 \cos n\pi_t dt$$

$$= \int_0^1 1 \cos n\pi_t dt$$

$$= \frac{1}{n\pi} [\sin n\pi_t]_0^1$$

$$= \frac{1}{n\pi} [\sin n\pi - \sin 0]$$

$$a_n = \frac{1}{n\pi} [0 - 0] = 0$$

$$\therefore a_n = 0$$

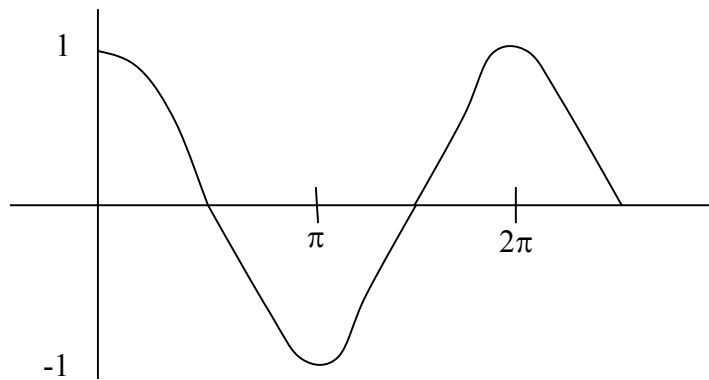
$$b_n = \frac{2}{T} \int_0^2 f(t) \sin n\pi_t dt$$

$$b_n = \frac{2}{2} \left(\int_0^1 1 \sin n\pi_t dt + \int_1^2 0 \sin n\pi_t dt \right)$$

$$b_n = \int_0^1 1 \sin n\pi_t dt + \int_1^2 0 \sin n\pi_t dt$$

$$= \int_0^1 1 \sin n\pi_t dt$$

$$\begin{aligned}
 &= \frac{1}{n\pi} [\cos n\pi t]_0^{-1} \\
 &= \frac{1}{n\pi} [\cos n\pi - \cos 0] \\
 &= -\frac{1}{n\pi} [\cos n\pi - 1]
 \end{aligned}$$



$$\cos[n\pi - 1] = \begin{cases} 1 & \text{Odd integer (1,3,5 \dots)} \\ 0 & \text{Even integer (2,4,6 \dots)} \end{cases}$$

$$b_n = -\frac{1}{n\pi} [\cos n\pi - 1]$$

$$b_n = -\frac{2}{n\pi}, \text{ where } n = 1,3,5, \dots$$

$$F(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + b_n \sin n\omega_0 t$$

$$= \frac{1}{2} + \sum_{n=1}^{\infty} \frac{2}{n\pi} \sin n\omega_0 t$$

$$F(t) = \frac{1}{2} + \frac{2}{\pi} \sin \omega_0 t + \frac{2}{3\pi} \sin 3\omega_0 t + \frac{2}{5\pi} \sin 5\omega_0 t$$

$$F(t) = \frac{1}{2} + \frac{2}{\pi} \sum_{n=1}^{\infty} \frac{1}{n} \sin n\omega_0 t \quad \text{where } n = 2k - 1$$

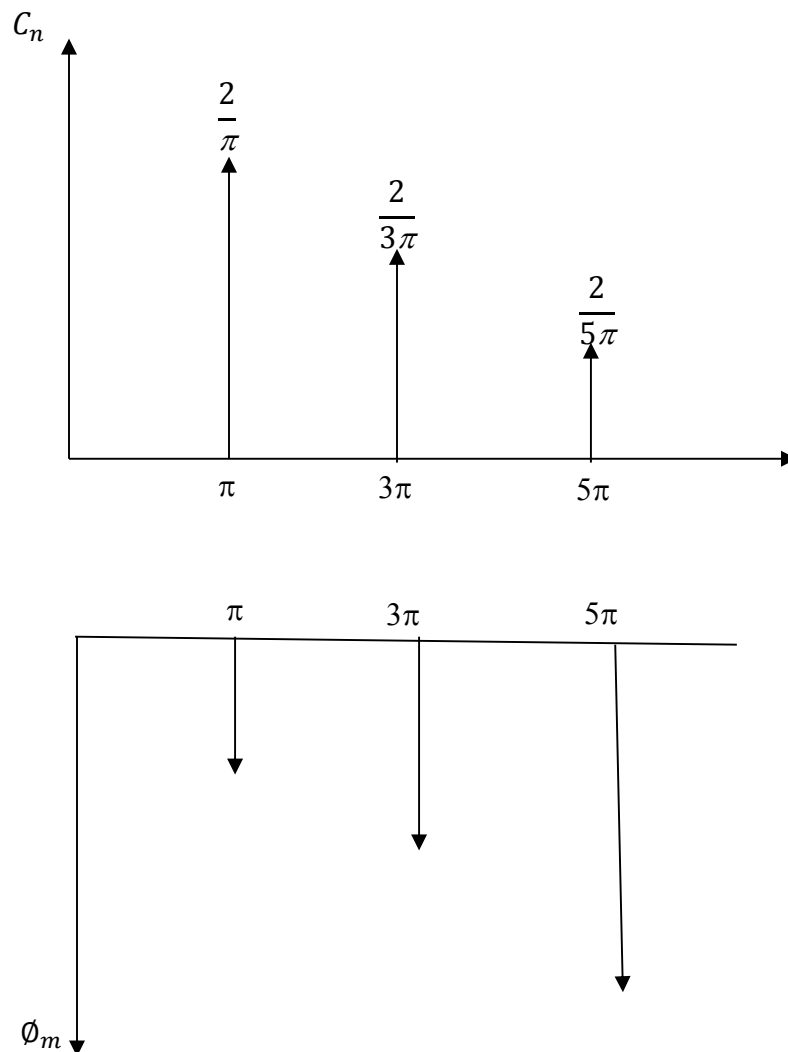
To obtain the amplitude and the phase spectra for the signal, we have:

$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\text{But } a_n = 0$$

$$\therefore C_n = \sqrt{b_n^2} = |b_n| = \begin{cases} \frac{2}{n\pi}, & n = \text{odd} \\ 0, & n = \text{even} \end{cases}$$

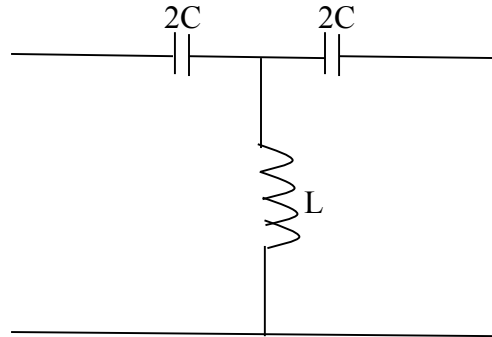
$$\phi_m = -\tan^{-1} \frac{b_n}{C_n} = \begin{cases} -90^\circ & \text{odd} \\ 0 & \text{even} \end{cases}$$



5a) Proof that $f_c = \frac{1}{4\pi\sqrt{LC}}$ in a high-pass prototype filter.

- b) Deduce the equation for the associated components
 c) A parallel resonant circuit has a $Q = 100$, inductor $L = 30\mu H$ with the resonant frequency as 15MHz. Calculate the power dissipated when the circuit is driven by a 10V r.m.s source.

Solution



$$Z_T = \sqrt{Z_A^2 + 2Z_A Z_B}$$

Where $Z_A = \frac{1}{j2\omega C}$ and $Z_B = j\omega L$

$$\therefore Z_T = \sqrt{\left(\frac{1}{j2\omega c}\right)^2 + 2\left(\frac{1}{j2\omega c}\right) * (j\omega L)}$$

$$= \sqrt{\left(\frac{1}{j^2 4\omega^2 c^2}\right) + \left(\frac{L}{c}\right)}$$

$$= \sqrt{\left(\frac{-1}{4\omega^2 c^2}\right) + \left(\frac{L}{c}\right)}$$

$$= \sqrt{\frac{L}{c} - \frac{1}{4\omega^2 c^2}}$$

At resonance, $\frac{L}{c} - \frac{1}{4\omega^2 c^2} = 0$

$$i.e \frac{L}{c} = \frac{1}{4\omega^2 c^2}$$

$$4\omega^2 c^2 L = C$$

$$\omega^2 = \frac{C}{4c^2 L}$$

$$\omega^2 = \frac{1}{4CL}$$

$$\omega = \frac{1}{\sqrt{4CL}}$$

$$\omega = \frac{1}{2\sqrt{CL}}$$

$$f = \frac{1}{4\pi\sqrt{CL}}$$

b) *Characteristics Impedance* $Z_o = \sqrt{\frac{L}{C}}$ (1)

$$f_c = \frac{1}{4\pi\sqrt{CL}} \quad (2)$$

From (1) $\sqrt{C} = \frac{\sqrt{L}}{Z_o}$

Substitute for C in (2)

$$f_c = \frac{1}{4\pi\sqrt{L} * \frac{\sqrt{L}}{Z_o}}$$

$$f_c = \frac{Z_o}{4\pi L}$$

$$L = \frac{Z_o}{4\pi f_c}$$

For C, we have:

$$\sqrt{L} = \sqrt{C} Z_o$$

$$f_c = \frac{1}{4\pi\sqrt{C} * \sqrt{C} Z_o}$$

$$= \frac{1}{4\pi C Z_o}$$

$$\therefore C = \frac{1}{4\pi f_c Z_o}$$

c)

Recall that $X_c = X_L$

$$\frac{1}{2\pi f C} = 2\pi f L$$

$$4\pi f^2 LC = 1$$

i.e $f = \frac{1}{2\pi\sqrt{LC}}$

Given $f = 15\text{MHz}$, and $L = 30\mu\text{H}$
 Substituting these values into the equation above, we obtain:

$$15 * 10^6 = \frac{1}{2 * 3.142\sqrt{30 * 10^{-6}C}}$$

$$i. e 15 * 10^6 * 2 * 3.142\sqrt{30 * 10^{-6}C} = 1$$

$$From\ the\ above, C = 3.752 * 10^{-12}F$$

$$Recall\ that\ Q_p = \frac{\omega_o C_p}{G}$$

$$G = \frac{\omega_o C_p}{Q_p}$$

$$= \frac{2\pi f C_p}{Q_p}$$

$$= \frac{2\pi 15 * 10^6 3.752 * 10^{-12}}{100}$$

$$G = 3.5363 * 10^{-6}S$$

$$R = \frac{1}{G}$$

$$\therefore R = 282.78k\Omega$$

$$Given\ V = 10V, \quad and\ R = 282.78k\Omega$$

$$I = \frac{10}{282.78 * 10^3}$$

$$= 3.536 * 10^{-5}A$$

$$P = I^2R$$

$$= (3.536 * 10^{-5})^2 282.78 * 10^3$$

$$= 3.53568 * 10^{-4}W$$

- 6a) As a Communication Consultant, a client was particularly interested in the term Filter; Explain the concepts and the reasons for its applications
- b) Design a bandpass filter that could capture frequencies in the range 250Hz and 3,000Hz with K=10 assuming R=20kΩ

Solution:

Filters are electrical or microwave devices designed to allow selected range of signal frequencies to pass, while obstructing those outside the range. They also reduce the possibility of interference between transmitted signals. They are used in a wide variety of applications

In addition, there are filters that do not filter any frequencies of a complex input signal, but just add a linear phase shift to each frequency component, thus contributing to a constant time delay. These are called all-pass filters. At high frequencies (>1MHz), all of these filters usually consists of passive component such as inductors (L), capacitors (C), and resistors (R). They are called RLC filters.

In the lower frequency range (1Hz-1MHz), the inductor values becomes very large and the inductor itself gets quite bulky, therefore making economical production difficult.

In these cases, active filters become important. Active filters are circuits that use an operational amplifier (op amp) as the active device in combination with some resistors and capacitors to provide an RLC-like filter performance at low frequencies

A generic type of filter is the band-pass filter: a filter with both high and low-frequency cutoffs. The channel filter in communications payload (subdivided into two parts-antennas and transponders) is a bandpass filter since it defines the usable bandwidth of the system eg, a transponder in satellite. Fig 1.8 shows four types of bandpass filters:

- (a) Flat passband, infinite out-of-band rejection; purely theoretical.
- (b) Butterworth with flat passband, slow roll-off. It has poor selectivity, rarely used in communications payloads. Butterworth filter response is called maximally flat in the bandpass.
- (c) Chebychev, passband ripple, good out-of-band rejection. The ripple is considered negligible compared with mismatch effects of other equipment. Commonly used in communications payloads.
- (d) Elliptic, a compromise, good selectivity and low ripple, but has a limited passband-to-stopband rejection ratio, hence not used in communications payloads.

7a) As a Communication Consultant, a client came for advice on the approach to transmit with minimal energy wastage and narrower frequency band; using the phase shift method, explain the generation of the SSB modulated signal.

- b) Show that the bandwidth at resonant frequency
- $$\omega_2 - \omega_1 = \frac{\omega_0}{Q}.$$

Sketch the amplitude characteristic showing the 3dB point.

8a) As a Communication Consultant, a client sought your advice on what type of modulation techniques he should adopt amongst AM, FM and PM. Explain their

differences, and what would you suggest?

- b) A technician observed an FM transmission having a constant-amplitude, variable frequency signal. He observed that the modulation index produced by *phase modulation* system as constant, but that produced by *frequency modulation* system is inversely proportional to the frequency of this signal. Explain to the technician:

- i) why this happened
- ii) what would the effect of white noise be on both systems
- iii) and what remedy would you recommend to ensure the transmitter transmits the required signals and the receiver recovers the original signals

- 9a) A carrier wave represented by $1.25\cos(2\pi f_c t)$ (volts) is amplitude modulated by a second wave represented by $0.5\cos(2\pi f_m t)$ (volts). If $f_c = 1\text{MHz}$ and $f_m = 1\text{kHz}$, Calculate the:

- iv) Lower and upper side frequencies
- v) Amplitude of the side frequencies
- vi) Fraction of the power transmitted in the sidebands

- 10 The figure below shows a time-variant signal $f(t)$ having a fundamental period T and amplitude A_0 .

- iii) Obtain the signal's Fourier transform
- iv) Under what condition would the signal be described as truly periodic?
- v) Estimate the average spectral power density

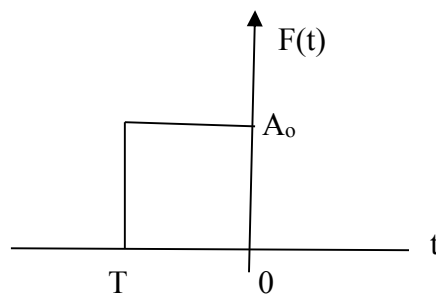


Fig 4 A pulse

11 The figure Q4 below is a time-variant signal $f(t)$ with the fundamental period and amplitude as stated in the diagram,

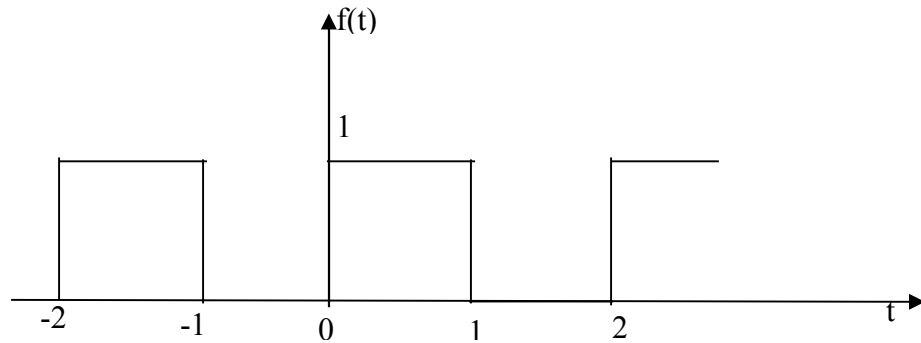


Fig. Q4;

- i) Obtain the signal's Fourier transform
 - ii) phase spectra of the signal.
- 12a) Proof that $f_c = \frac{1}{4\pi\sqrt{LC}}$ in a high - pass prototype filter.
- b) Deduce the equation for the associated components
 - c) A parallel resonant circuit has a $Q = 100$, inductor $L = 30\mu H$ with the resonant frequency as 15MHz. Calculate the power dissipated when the circuit is driven by a 10V r.m.s source.
- 13a) As a Communication Consultant, your client asks you a series of question to clear his confused mind. Why do
- i) Why do we use filters
 - ii) Why is a class of filters called colloquially as “maximally flat”?
 - iii) What type of filters behaves this way?
- b) Two singers have a vocal-range of frequencies and both compliments each other. As an engineer, develop a filter that would capture the essence of their vocal ranges. State all assumptions made.
- 14a) As a Communication Consultant, a client was particularly interested in the term Filter; Explain the concepts and the reasons for its applications
- b) Design a bandpass filter that could capture frequencies in the range 250Hz and 3,000Hz with $K=10$ assuming $R=20k\Omega$
- 15a) Give examples of the main applications of autocorrelation and cross-correlation functions
- b) A discrete-time signal $s_1(t)$ is convolved with another discrete-time signal $s_2(t)$; that is $s_1(t)*s_2(t)$. State all assumptions made, and express the resulting expression $y(t)$ in the frequency domain
 - c) Explain briefly the principle of Delta modulation scheme. Give an example of the source of error and how it can be corrected
- 16 Explain briefly the processes listed below:
- i) Quantization

- ii) Pulse amplitude modulation 1
- iii) Pulse code modulation
- iv) Nyquist rate
- v) Sampling interval

17 Determine the spectrum of a signal

$$s(t) = \begin{cases} 1 & |t| < \frac{\tau}{2} \\ 0 & |t| \geq \frac{\tau}{2} \end{cases}$$

Solution

Spectrum = Fourier transform

Substitute (i) directly in (1.11) to have

$$\begin{aligned} S(f) &= \int_0^{\tau/2} 1e^{-j2\pi ft} \\ &= \frac{1}{j2\pi f} [1 - e^{-j\pi f\tau}] \end{aligned} \tag{ii}$$

We can manipulate the [.] term as follows. Note that

$$\begin{aligned} \sin(a) &= \frac{e^{ja} - e^{-ja}}{j2} \\ &= e^{ja} \frac{1 - e^{-j2a}}{j2} \end{aligned} \tag{iii}$$

If we make $a = \frac{1}{2}\pi f\tau$, we can write

$$e^{-j\pi f\tau/2} \sin\left(\frac{1}{2}\pi f\tau\right) = \left[\frac{1 - e^{-j\pi f\tau}}{2j}\right] \tag{iv}$$

Comparing (iv) with (ii), and multiplying the numerator (top) and denominator (bottom) by $\tau/2$, we can write the signal spectrum as

$$\begin{aligned} S(f) &= \frac{\tau e^{-\pi f\tau/2} \sin\left(\frac{\pi f\tau}{2}\right)}{2 \frac{\pi f\tau}{2}} \\ &= \frac{\tau e^{-\pi f\tau/2}}{2} \text{sinc}\left(\frac{\pi f\tau}{2}\right) \end{aligned} \tag{v}$$

18 Obtain the Fourier transform of the signal

$$s(t) = \begin{cases} \alpha \cos\left(\frac{2\pi t}{\Delta}\right) & -\frac{T}{2} \leq t \leq \frac{T}{2} \\ 0 & > \frac{T}{2} \end{cases}$$

which is a "periodic" signal in the interval $-\frac{T}{2}$ to $\frac{T}{2}$, where α and Δ correspond to the amplitude of the signal and a scaling factor. Also, obtain the Fourier transform of the signal if it is truly periodic.

Solution

Using the definition (1.11) and substitute (vi) in it, the signal's Fourier transform is

$$\begin{aligned} S(f) &= \int_{-T/2}^{T/2} \alpha \cos\left(\frac{2\pi t}{\Delta}\right) e^{j2\pi ft} dt \\ &= \frac{\alpha}{2} \left\{ T \frac{\sin \pi T \left[f - \left(\frac{1}{\Delta} \right) \right]}{\pi T \left[f - \left(\frac{1}{\Delta} \right) \right]} + T \frac{\sin \pi T \left[f + \left(\frac{1}{\Delta} \right) \right]}{\pi T \left[f + \left(\frac{1}{\Delta} \right) \right]} \right\} \end{aligned} \quad \text{(vii)}$$

As T tends to infinity, the signal s(t) becomes a truly periodic signal, periodic for all time, while the transform S(f) tends to

$$S(f) = \frac{\alpha}{2} \left[\delta\left(f - \frac{1}{\Delta}\right) + \delta\left(f + \frac{1}{\Delta}\right) \right] \quad \text{(viii)}$$

From (viii) we can conclude that the Fourier transform of a truly periodic (infinite extent) cosine wave consists of a delta function of area $\frac{\alpha}{2}$ centered at frequency

$$f = \pm \frac{1}{\Delta}$$

:19 Find the autocorrelation of a periodic waveform $v(t)$ defined by

$$v(t) = \sum_{n=-\infty}^{\infty} v_n e^{j2\pi nt/T_0}$$

Solution

Using correlation integral of (1.36), we can write

$$R(\tau) = \frac{1}{T_0} \int_{-T_0/2}^{T_0/2} \left(\sum_{n=-\infty}^{\infty} v_n e^{j2\pi n(t+\tau)/T_0} \right) \left(\sum_{m=-\infty}^{\infty} v_m e^{j2\pi mt/T_0} \right) dt \quad (3ii)$$

The order of integration and summation in (3ii) can be interchanged. So, we can represent the m, n terms integrand by, say,

$$I_{m,n} = \frac{1}{T_0} e^{j2\pi n\tau/T_0} \int_{-T_0/2}^{T_0/2} v_m v_n e^{j2\pi(m+n)t/T_0} dt \quad (3iii)$$

By normal integration process, the solution to (iii) is

$$\begin{aligned} I_{m,n} &= v_m v_n e^{j2\pi n\tau/T_0} \frac{\sin \pi(m+n)}{\pi(m+n)} \\ &= v_m v_n e^{j2\pi n\tau/T_0} \text{sinc} \pi(m+n) \end{aligned} \quad (3iv)$$

Since m, n are integers, (i.e., $m = 0, 1, 2, \dots$; $n = 0, 1, 2, \dots$), numerical values can be obtained for (3iv) as follows. When

$$\begin{aligned} m = n = 0 & \quad \text{sinc}(0) = 1 \\ m = n = 1 & \quad \text{sinc}(2\pi) = 0 \\ m = n = 2 & \quad \text{sinc}(4\pi) = 0 \end{aligned} \quad (3v)$$

The above iterative solution has shown that $I_{m,n}$ has a non-zero value when $(m+n) = 0$, meaning that $m = -n$. Substitute $m = -n$ in (3iv), we have

$$I_{m,n} = v_m v_n e^{j2\pi n\tau/T_0} \quad (3vi)$$

Finally, the autocorrelation becomes

$$\begin{aligned}
R(\tau) &= \sum_{n=-\infty}^{\infty} v_{-n} v_n e^{j2\pi n\tau/T_o} \\
&= \sum_{n=-\infty}^{\infty} |v_n|^2 e^{j2\pi n\tau/T_o}
\end{aligned} \tag{3vii}$$

$$R(\tau) = |v_o|^2 + 2 \sum_{n=1}^{\infty} |v_n|^2 e^{j2\pi n\tau/T_o} \tag{3viii}$$

We note that the Fourier transforms, and using equation (1.11), expression (3viii) can be expressed as

$$\mathfrak{S}[R(\tau)] = \int_{-\infty}^{\infty} \left(\sum_{n=-\infty}^{\infty} |v_n|^2 e^{j2\pi n\tau/T_o} \right) e^{-j2\pi f\tau} d\tau \tag{3ix}$$

Interchanging the order of integration and summation, we can arrange (3x) as

$$\mathfrak{S}[R(\tau)] = \sum_{n=-\infty}^{\infty} |v_n|^2 \int_{-\infty}^{\infty} e^{-j2\pi \left(f - \frac{n}{T_o}\right)\tau} d\tau \tag{3x}$$

By Fourier frequency translation, equation (3xi) appears to be frequency translated.

Hence,

$$\mathfrak{S}[R(\tau)] = \sum_{n=-\infty}^{\infty} |v_n|^2 \delta\left(f - \frac{n}{T_o}\right) \tag{3xi}$$

This expression can be compared with the Fourier expression for periodic waveform:

$$G(f) = \mathfrak{S}[R(\tau)] \tag{3xii}$$

Of course, conversely,

$$R(\tau) = \mathfrak{S}^{-1}[G(f)] \tag{3xiii}$$

Simply, *the power spectral density and the correlation function of a periodic waveform are a Fourier transform pair*. This relationship is called the Wiener-Khintchine relationship¹

¹ Wiener, N (1930). Generalized harmonic analysis. *Acta Math*, **55**, p.117

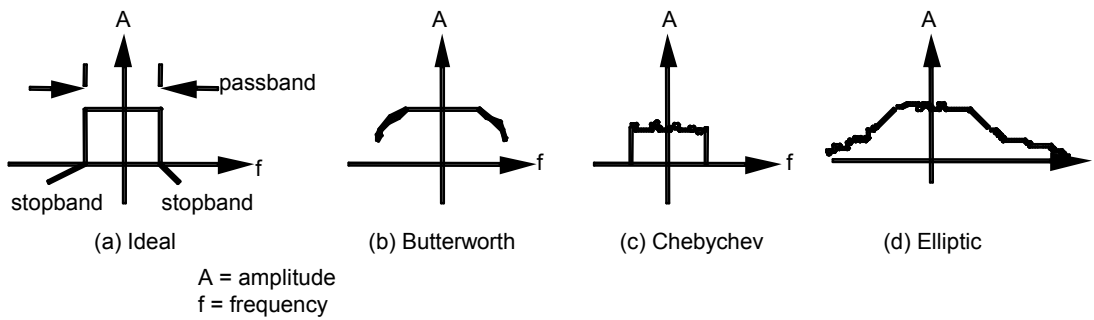


Figure 1.8. Types of bandpass filter.

Bessel filter has a response that is called maximally flat delay in magnitude: it is also known to give a linear phase response. Bessel filter is not considered in this text/lecture.

Note that:

- (i) The input and output channel filters may be either bandpass or low pass filter. Low pass filter has a high frequency cutoff allowing only low frequency signals to pass, while the high-pass filter is the opposite of low-pass filter.
- (ii) Filters are often installed in the form of input/output multiplexers or diplexers. (A *diplexer* is a 2-channel multiplexer. A *multiplexer* is a device for combining or separating signal frequencies. In actual fact, a *multiplexer* is a multichannel filter.
- (iii) The channel output filter has a narrower passband than the input filter. It is this output channel filter that defines the *useable bandwidth* of the device or system; an important quantity as far as the user of the service is concerned.

A filter is a system that changes the frequency characteristics of its input signal.

Examples of filtering operations:

1. Noise suppression in radio signals, sensors, etc.
2. Enhancement of certain frequency ranges in music and images (edge enhancement), etc.
3. Bandwidth limiting in sampling radio, TV, etc.
4. Removal or attenuation of specific frequencies such as DC component or power line interference.

5. Special operations: differentiation, integration.

20 Given R and frequency specified,

Since $\omega_1 = \frac{1}{RC_2}$, we obtain

$$C_2 = \frac{1}{R\omega_1} = \frac{1}{2\pi f_1 R} = \frac{1}{2\pi * 250 * 20 * 10^3}$$

$$= 31.83nF$$

Similarly, $\omega_2 = \frac{1}{RC_1}$

$$C_1 = \frac{1}{R\omega_2} = \frac{1}{2\pi f_2 R} = \frac{1}{2\pi * 3000 * 20 * 10^3}$$

$$= 2.65nF$$

From previous analysis:

$$\frac{R_f}{R_i} = K \frac{\omega_1 + \omega_2}{\omega_2} = K \frac{f_1 + f_2}{f_2}$$

$$10 \frac{3250}{3000} = 10.83$$

If we select $R_i = 10k\Omega$, then $R_f = R_i * 10.83 = 108.3k\Omega$